CBSE ACADEMY PLUS

Unit 6

Application of Derivatives

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The function $f(x) = x^3 + 3x$ is increasing in interval
(a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) R (d) (0, 1)

2. In a sphere of radius 'r', a right circular cone of height 'h' having maximum curved surface area is inscribed. The expression for the square of curved surface of cone is
(a) $2\pi^2rh(2rh + h^2)$ (b) $\pi^2hr(2rh + h^2)$ (c) $2\pi^2r(2rh^2 - h^3)$ (d) $2\pi^2r^2(2rh - h^2)$

3. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval
(a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, 2)$ (c) $(-\infty, 2] \cup [3, \infty)$ (d) $(3, \infty)$

4. A function $f : R \to R$ is defined as $f(x) = x^3 + 1$. Then the function has
(a) no minimum value (b) no maximum value
(c) both maximum and minimum values (d) neither maximum value nor minimum value

5. The point(s) on the curve $y = x^2$, at which y-coordinate is changing six times as fast as x-coordinate is/are
(*a*) (2, 4) (*b*) (3, 9) (*c*) (3, 9), (9, 3) (*d*) (6, 2)

6. The edge of a cube is increasing at the rate of 0.3 cm/s, the rate of change of its surface area when edge is 3 cm is
(a) 10.8 cm (b) 10.8 cm^2 (c) 10.8 cm^2/s (d) 10.8 cm/s

7. The function $f(x) = 4 - 3x + 3x^2 - x^3$, $x \in R$ is
(a) decreasing function (b) increasing function
(c) strictly increasing on R (d) neither increasing nor decreasing on R

8. If at x = 1, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2]. Then the value of a is
(a) 124 (b) –124 (c) 120 (d) –120

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The maximum value of the function $f(x) = x^5$, $x \in [-1, 1]$, is attained at its critical point, $x = 0$.
Reason (R): The maximum value of a function can only occur at points where derivative is zero.

10. **Assertion (A):** The function $f(x) = x^3 - 12x$ is strictly increasing in $(-\infty, -2) \cup (2, \infty)$.
Reason (R): For strictly increasing function $f'(x) > 0$.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. For what values of x is the rate of increase of $x^3 - 5x^2 + 5x + 8$ is twice the rate of increase of x?

12. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$ is decreasing for all $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

13. A spherical balloon is being inflated by pumping in 16 cm^3/s of gas. At the instant when balloon contains 36π cm^3 of gas, how fast is its radius increasing?

14. Find the least value of a such that the function $f(x) = x^2 + ax + 1$ is strictly increasing on $[1, 2]$.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is (*i*) strictly increasing (*ii*) strictly decreasing

16. Find the absolute maximum value and the absolute minimum value for the function $f(x) = 4x - \frac{1}{2}x^2$, in the given interval $x \in \left[-2, \frac{9}{2}\right]$.

17. A man 1.6 m tall walks at the rate of 0.5 m/s away from a lamp post, 8 metres high. Find the rate at which his shadow is increasing and the rate with which the tip of shadow is moving away from the pole.

SECTION – D

Questions 18 carry 5 marks.

18. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides, what is the cost of least expensive tank?

OR

Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. **Case-Study 1:**
The use of electric vehicles will curb air pollution in the long run.

The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V:

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions:

(a) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.

(b) Prove that the function V(t) is an increasing function.

20. Case-Study 2:

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)